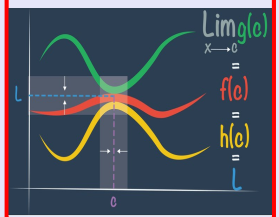
Calculus I

Lecture 20



Feb 19-8:47 AM

Find $\lim\limits_{x\to\infty} \dfrac{1-x^2}{x^3-x+1} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x^3}\to 0-\frac{1}{x}\to 0}{1-\frac{1}{x^2}\to 0+\frac{1}{x^3}\to 0} = \dfrac{0}{1} = \boxed{0}$

Divide by x^3

Find $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}+x^2}{2x-x^2} = \lim\limits_{x\to\infty} \dfrac{\frac{\sqrt{x}}{x^2}+\frac{x^2}{x^2}\to 1}{\frac{2x}{x^2}-\frac{x^2}{x^2}\to 1}$

Divide by x^2

as $x\to\infty$ $x^2=\sqrt{x^4}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{\sqrt{x}}{\sqrt{x^4}}+1}{\frac{2}{x}-1}$

$= \lim\limits_{x\to\infty} \dfrac{\sqrt{\frac{1}{x^3}}\to 0+1}{\frac{2}{x}\to 0-1} = \dfrac{1}{-1} = \boxed{-1}$

Oct 2-7:25 AM

Find $\lim\limits_{x\to\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$ $\quad \dfrac{\infty}{\infty}$ I.F.

$$\frac{\sqrt{9x^6-x}}{x^3+1} \approx \frac{\sqrt{9x^6}}{x^3} \approx \frac{3x^3}{x^3} = 3$$

Divide by x^3 $\qquad$ as $x\to\infty$, $x^3 = \sqrt{x^6}$

$$\lim_{x\to\infty} \frac{\frac{\sqrt{9x^6-x}}{x^3}}{\frac{x^3+1}{x^3}} = \lim_{x\to\infty} \frac{\sqrt{\frac{9x^6-x}{x^6}}}{\frac{x^3+1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{9x^6}{x^6} - \frac{x}{x^6}}}{\frac{x^3}{x^3} + \frac{1}{x^3}} = \lim_{x\to\infty} \frac{\sqrt{9 - \cancel{\frac{1}{x^5}}^{\to 0}}}{1 + \cancel{\frac{1}{x^3}}^{\to 0}} = \frac{\sqrt{9}}{1} = \boxed{3}$$

what if $x \to -\infty$?

Find $\lim\limits_{x\to-\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$ $\quad \dfrac{\infty}{-\infty}$ I.F.

$$\boxed{\begin{array}{l} \text{as } x\to\infty,\ x = \sqrt{x^2} \\ \text{as } x\to-\infty,\ x = -\sqrt{x^2} \end{array}}$$

$$\frac{\sqrt{9x^6-x}}{x^3+1} \approx \frac{\sqrt{9x^6}}{x^3} \approx \frac{-3x^3}{x^3} = -3$$

Divide by x^3 $\qquad$ as $x\to-\infty$, $x^3 = -\sqrt{x^6}$

$$\lim_{x\to-\infty} \frac{\frac{\sqrt{9x^6-x}}{x^3}}{\frac{x^3+1}{x^3}} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{9x^6-x}{x^6}}}{\frac{x^3+1}{x^3}}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{9x^6}{x^6} - \frac{x}{x^6}}}{\frac{x^3}{x^3} + \frac{1}{x^3}} = \lim_{x\to} \frac{-\sqrt{9 - \cancel{\frac{1}{x^5}}^{\to 0}}}{1 + \cancel{\frac{1}{x^3}}^{\to 0}} =$$

$$= \frac{-\sqrt{9}}{1} = \boxed{-3}$$

Evaluate $\lim_{x \to -\infty} \frac{x - 4}{\sqrt{x^2 + 4}}$

as $x \to -\infty$

$x = -\sqrt{x^2}$

$\frac{x-4}{\sqrt{x^2+4}} \approx \frac{x}{\sqrt{x^2}} = \frac{x}{-x}$ $\quad \frac{-\infty}{\infty}$

$\lim_{x \to -\infty} \frac{\frac{x}{x} - \frac{4}{x}}{\frac{\sqrt{x^2+4}}{x}} = \lim_{x \to -\infty} \frac{1 - \frac{4}{x}}{\frac{\sqrt{x^2+4}}{-\sqrt{x^2}}}$

$= \lim_{x \to -\infty} \frac{1 - \frac{4}{x} \to 0}{-\sqrt{\frac{x^2}{x^2} \to 1 + \frac{4}{x^2} \to 0}} = \frac{1}{-1} = \boxed{-1}$

Oct 2-7:47 AM

$\lim_{x \to \infty} \left(\sqrt{x^2+4x} - x \right) = \infty - \infty$

$\sqrt{x^2+4x} \approx \sqrt{x^2} = x$

$\sqrt{x^2+4x} - x \approx x - x$

I.F. $\frac{0}{0}, \frac{\infty}{\infty}, \infty \cdot 0, \infty - \infty$

$\lim_{x \to \infty} \frac{(\sqrt{x^2+4x} - x)(\sqrt{x^2+4x} + x)}{(\sqrt{x^2+4x} + x)} = \lim_{x \to \infty} \frac{\cancel{x^2} + 4x - \cancel{x^2}}{\sqrt{x^2+4x} + x}$

$= \lim_{x \to \infty} \frac{4x}{\sqrt{x^2+4x} + x}$ $\quad \frac{\infty}{\infty}$ I.F.

$\frac{\sqrt{x^2+4x}}{x} = \frac{\sqrt{x^2+4x}}{\sqrt{x^2}} = \sqrt{\frac{x^2+4x}{x^2}}$

Divide by x, Keep in mind as $x \to \infty$, $x = \sqrt{x^2}$

$= \lim_{x \to \infty} \frac{4}{\sqrt{\frac{x^2+4x}{x^2}} + 1} = \lim_{x \to \infty} \frac{4}{\sqrt{1 + \frac{4}{x} \to 0} + 1}$

$= \frac{4}{2} = \boxed{2}$

Oct 2-7:54 AM

Find $f'(x)$ for $f(x) = x^3 - 3x$, then find $f'(2)$.

$f(2) = 2^3 - 3(2) = 8 - 6 = 2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

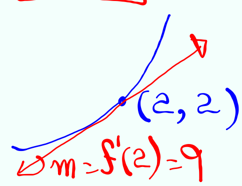
$$= \lim_{h\to 0} \frac{(x+h)^3 - 3(x+h) - x^3 + 3x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{3x} - 3h - \cancel{x^3} + \cancel{3x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + \cancel{3xh} + \cancel{h^2} - 3)}{\cancel{h}} = \boxed{3x^2 - 3}$$

$f'(2) = 3(2^2) - 3 = 3\cdot 4 - 3 = \boxed{9}$

$y - 2 = 9(x - 2)$ $\boxed{y = 9x - 16}$



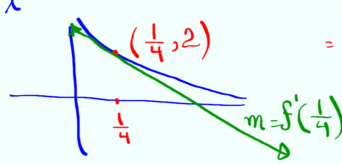
Oct 2-8:02 AM

Find equation of the tan. line to the graph $f(x) = \frac{1}{\sqrt{x}}$ at $x = \frac{1}{4}$.

$f(\frac{1}{4}) = \frac{1}{\sqrt{\frac{1}{4}}} = 2$

$\lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$

$\lim_{x\to 0^+} \frac{1}{\sqrt{x}} = \infty$



$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

LCD $= \sqrt{x+h}\sqrt{x}$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}} = \lim_{h\to 0} \frac{\overbrace{(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}^{\cancel{x} - \cancel{x} - \cancel{h}\ \ -1}}{\cancel{h}\sqrt{x+h}\sqrt{x}(\sqrt{x} + \sqrt{x+h})}$$

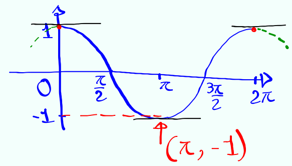
$$= \lim_{h\to 0} \frac{-1}{\sqrt{x+h}\sqrt{x}(\sqrt{x} + \sqrt{x+h})} = \frac{-1}{\sqrt{x^2}\cdot 2\sqrt{x}}$$

$f'(x) = \boxed{\frac{-1}{2x\sqrt{x}}}$ $m = f'(\frac{1}{4}) = \frac{-1}{2\cdot\frac{1}{4}\cdot\sqrt{\frac{1}{4}}} = \boxed{-4}$

$y - 2 = -4(x - \frac{1}{4}) \rightarrow \boxed{y = -4x + 3}$

Oct 2-8:11 AM

find points on the interval $[0, 2\pi]$ where $f(x) = \cos x$ has horizontal tan. line.



$m = 0$

$f'(x) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\cos x (\cosh - 1)}{h} - \frac{\sin x \sinh}{h} \right]$$

$$= \cos x \underbrace{\lim_{h \to 0} \frac{\cosh - 1}{h}}_{\to 0} - \sin x \underbrace{\lim_{h \to 0} \frac{\sinh}{h}}_{\to 1}$$

$f'(x) = -\sin x$

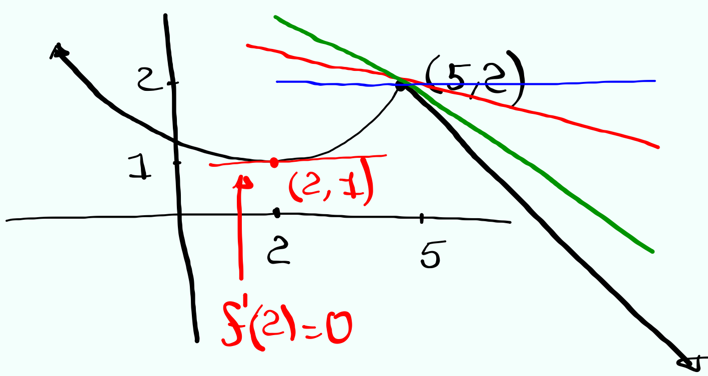
Solve $f'(x) = 0$ $\quad -\sin x = 0 \quad \sin x = 0$

$0, \pi, 2\pi$

Endpoints

only choice

Oct 2-8:22 AM



All are tan. lines

$f'(5)$ does not exist.

Oct 2-8:32 AM

Evaluate $\lim\limits_{x\to\infty} \dfrac{4x - x^2}{\sqrt{4x^2+6}} = \dfrac{-\infty}{\infty}$ I.F.

$x^2 = \sqrt{x^4}$

as $x\to\infty$

Divide by x^2

$$\lim_{x\to\infty} \frac{\frac{4x}{x^2} - \frac{x^2}{x^2}}{\sqrt{\frac{4x^2+6}{x^4}}} = \lim_{x\to\infty} \frac{\cancel{\frac{4}{x}}^{\to 0} - 1}{\sqrt{\cancel{\frac{4}{x^2}}^{\to 0} + \cancel{\frac{6}{x^4}}^{\to 0}}}$$

$= \dfrac{-1}{0}$ undefined

Use a graphing Software and graph $f(x) = \dfrac{4x - x^2}{\sqrt{4x^2+6}}$ and explore as $x\to\infty$

Oct 1-7:50 AM